Noise and deviation effects in a bichromatic Raman white light cavity

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We analyze the effects of noise and parameter deviations in a bichromatic Raman type white light cavity, with potential applications in precision measurements such as gravitational wave detection. The results show that the dispersion variation induced by parameter deviation can be controlled within 10^{-4}. The laser phase noise decreases the dispersion magnitude while the amplitude noise increases it. Although we can always adjust the parameters to satisfy the white light condition, both noises make the cavity transmission curve uneven.

I. INTRODUCTION

In an Fabry-Perot cavity the round-trip phase delay is proportional to the frequency. Thus only certain discrete frequencies can be exactly resonant. If the cavity is filled with a medium which provides a negative dispersion and cancels the frequency dependence of the phase delay, a continuous range of spectrum can be resonant at the same time. Such a cavity is named as white light cavity (WLC) [1]. For precision measurements such as gravitational wave detection [2, 3, 4] and ring laser gyroscopes [5], the high sensitivity requires a high finesse Fabry-Perot cavity, at the price of a reduced bandwidth. WLC provides an effective way to increase the bandwidth and solves this dilemma.

The dispersion requirement for the medium is $\partial_\nu n = -1/\nu$, where the refractive index n is a function of the frequency ν. This is the so called λ-compensation or white light condition. A lot of systems are able to provide negative dispersion with small absorption or even gain [6, 7]. For example, for two-level atoms driven by a strong resonant field [8, 9], the probe dispersion is negative around the resonance. A variation of this scheme is the degenerate two-level system [10], in which there are two degenerate ground levels. Both the resonant drive field and the probe field interact with the two transitions simultaneously. The advantage of this scheme is that it does not require a very strong drive field. For a Λ system with a bichromatic drive field far from resonance [11, 12], the probe field experiences a gain-doublet and the dispersion is negative at the center. Another system is double-Λ system in which the driving field interacts with the transitions from both ground levels to one of the excite level, and the probe field interacts with the transitions from both ground levels to the other excite level [2]. Wicht et al. analyzed and compared these systems in detail [7]. Recently Savchenkov and co-workers demonstrated white light whispering gallery mode resonators [13]. For a resonator thick enough the modal spectrum becomes essentially continuous and the high quality factor is frequency-independent.

The idea of the gain-doublet scheme is proposed by Steinberg and Chiao during their persuit of superluminal pheonomena [14]. Wang et al. first realized it experimentally in a Λ system [11, 12]. Due to the negative dispersion, the group velocity can be superluminal or even negative. The ideal case of infinite group velocity is equivalent to the white light condition. The ability of this system to achieve the white light condition has been investigated by measuring the dispersion using a heterodyne technique [15], and by measuring the transmission spectrum [16]. A broadband cavity response has been observed.

In order to satisfy the white light condition we need to choose the parameters carefully. However, there are always deviations from the ideal values [2] and statistical noise. In this paper we discuss the effects of parameter deviations and laser phase and amplitude noises in the bichromatic Raman system.

II. PARAMETER DEPENDENCE OF THE SUSCEPTIBILITY

The level structure of the bichromatic Raman system is shown in Fig. 1. There are two drive fields with frequencies ν_1 and ν_2 and Rabi frequencies Ω_1 and Ω_2, respectively. They are far detuned from the transition $|a\rangle \leftrightarrow |c\rangle$ with the detunings $\Delta_0 + \Delta$ and $\Delta_0 - \Delta$, where $\Delta = (\nu_1 - \nu_2)/2$ and $\Delta_0 = \omega_{ac} - (\nu_1 + \nu_2)/2$. The probe field frequency ν scans across the two Raman transitions. Such a gain doublet provides the negative dispersion at the center.

The susceptibility of the probe field can be written as [12]

$$\chi(\nu) = \frac{M_1}{(\nu - \nu_0 - \Delta) + i\Gamma} + \frac{M_2}{(\nu - \nu_0 + \Delta) + i\Gamma}, \quad (1)$$

where $\nu_0 = \frac{1}{2}(\nu_1 + \nu_2) - \omega_{bc}$ is the probe central frequency, $M_j = N(|\mu_{ab} \cdot \hat{e}|^2/2\hbar\epsilon_0)(|\Omega_j|^2/\Delta_0^2), (j = 1, 2)$, and Γ is the Raman transition line broadening. Usually we have
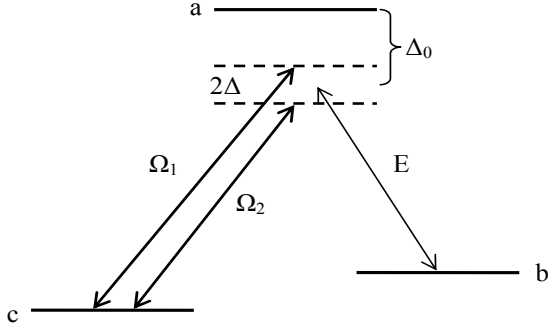
FIG. 1: The scheme of the bichromatic Raman system. The pump fields are far detuned from the single photon transition $|a\rangle \leftrightarrow |c\rangle$ and provides a gain-doublet for the probe field.
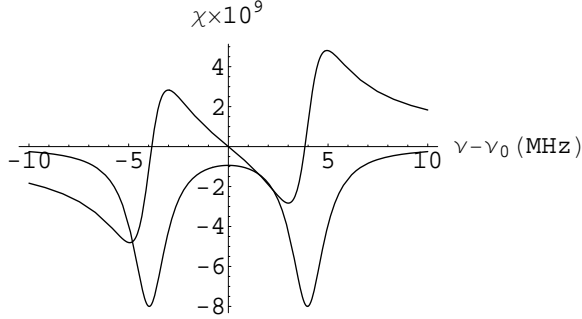

FIG. 2: The probe susceptibility of a typical bichromatic Raman system. The symmetric curve is the imaginary part and the anti-symmetric curve is the real part. The dispersion inside the gain-doublet is negative.

$M_1 \cong M_2 = M$ to get the symmetrical gain peaks. Typical susceptibility curves are shown in Fig.2.

From the susceptibility, we can determine the refractive index n and the absorption coefficient α. The resulting expressions at the central frequency are

$$n \cong 1 + \frac{1}{2}\chi' = 1 + \frac{1}{2}\frac{(-M_1 + M_2)\Delta}{\Delta^2 + \Gamma^2}, \qquad (2)$$

$$\alpha \cong \frac{\nu_0}{2c}\chi'' = -\frac{\nu_0}{2c}\frac{(M_1 + M_2)\Gamma}{\Delta^2 + \Gamma^2}, \qquad (3)$$

where χ' and χ'' are the real and imaginary parts of the susceptibility χ. The dispersion at ν_0 is given by

$$\partial_\nu n = -\frac{M_1 + M_2}{2}\frac{(\Delta^2 - \Gamma^2)}{(\Delta^2 + \Gamma^2)^2}. \qquad (4)$$

By choosing the parameters carefully we can have a dispersion equal to $-1/\nu_0$. The white light condition is then satisfied.

In order to analyze the effect of the parameter deviations, we note that M_j is proportional to both the pump field intensity I_j and the number density N. Therefore the deviations of I_j or N lead to the variation of the absorption, dispersion and refractive index as

$$\frac{\delta(\partial_\nu n)}{\partial_\nu n} = \frac{\delta\alpha}{\alpha} = \frac{\delta M_1 + \delta M_2}{2M}, \qquad (5)$$

$$\delta n = \frac{1}{2}\frac{(-\delta M_1 + \delta M_2)\Delta}{\Delta^2 + \Gamma^2}. \qquad (6)$$

The two intensity deviations can be independent from each other. From the proportionality between M_j and I_j we get

$$\frac{\delta(\partial_\nu n)}{\partial_\nu n} = \frac{\delta\alpha}{\alpha} = \frac{1}{2}(\frac{\delta I_1}{I_1} + \frac{\delta I_2}{I_2}). \qquad (7)$$

It is easier to keep the white light condition if the relative intensity deviations of the two driving fields are of opposite signs.

On the other hand, number density deviation affects M_1 and M_2 simultaneously. Therefore

$$\frac{\delta(\partial_\nu n)}{\partial_\nu n} = \frac{\delta\alpha}{\alpha} = \frac{\delta N}{N}. \qquad (8)$$

From Eq.(6), the refractive index does not change under the number density deviation.

Next we consider the effect of drive frequency deviation. If the frequency ν_1 is changed by the amount $\delta\nu_1$ and ν_2 is changed by $\delta\nu_2$, the susceptibility would become

$$\chi(\nu) = \frac{M}{(\nu - \nu_0 - \Delta - \delta\nu_1) + i\Gamma} + \frac{M}{(\nu - \nu_0 + \Delta - \delta\nu_2) + i\Gamma}, \qquad (9)$$

From the susceptibility we can derive

$$\delta n = \frac{M(\Delta^2 - \Gamma^2)}{2(\Delta^2 + \Gamma^2)^2}(\delta\nu_1 + \delta\nu_2), \qquad (10)$$

$$\delta\alpha = \frac{\nu_0 M\Delta\Gamma}{c(\Delta^2 + \Gamma^2)^2}(\delta\nu_1 - \delta\nu_2), \qquad (11)$$

$$\frac{\delta(\partial_\nu n)}{\partial_\nu n} = -\frac{\Delta(\Delta^2 - 3\Gamma^2)}{\Delta^4 - \Gamma^4}(\delta\nu_1 - \delta\nu_2). \qquad (12)$$

We consider the parameters from Ref. [16], i.e., $\Delta = 3.97 MHz$, $\Gamma = 1MHz$ and $\lambda = 780nm$. On substituting these values into the above expressions we obtain $\delta n = 2.07 \times 10^{-16}(s)(\delta\nu_1 + \delta\nu_2)$, $\delta\alpha = 8.97 \times 10^{-10}(s/m)(\delta\nu_1 - \delta\nu_2)$, and $\delta(\partial_\nu n)/\partial_\nu n = -2.05 \times 10^{-7}(s)(\delta\nu_1 - \delta\nu_2)$. Compared to the double-Λ system [2], the dependence of refractive index on the drive frequency is smaller in our system. However the dependence of the dispersion and the absorption on the drive

frequency are much larger. If these two drive fields are generated from the same laser then the frequency deviations would be the same and there is no dispersion and absorption variations in this case.

Based on the same argument as in Ref. [2] we conclude that the variation results in bichromatic Raman type white light cavity can be controlled within 10^{-4}. So in theory the white light cavity linewidth could be 10^4 times broader than empty cavity. But of course one has to include the other imperfect effects such as the nonlinear shape of the dispersion curve, etc.

III. EFFECT OF LASER PHASE AND AMPLITUDE NOISE

In the previous section, we calculated the effect of parameter deviations, or in a more strict sense, the deviation of the expectation value. Here we consider the noise effect from the drive fields. In other words, the expectation values may have satisfied the white light condition, but the random fluctuation of the laser phase and amplitude will nevertheless modify the dispersion. The phase noises account for the finite linewidth of the drive fields and the amplitude noises are responsible for the intensity fluctuations. We calculate the effect of these noise sources independentally. For simplicity sake we assume that the separation between the two Raman peaks is much larger than the Raman linewidth and therefore we can calculate the two Raman transitions independentally.

Following the expressions in Ref. [12], the effective Hamiltonian for the system can be written as

$$
\begin{aligned}
\hat{H} &= \hat{H}_0 + \hat{H}_I = -\hbar\omega_{ab} |b\rangle \langle b| - \hbar\omega_{ac} |c\rangle \langle c| \\
&- \hbar\Omega_p e^{-i\nu t} |a\rangle \langle b| - \hbar\Omega_1 e^{-i\nu_1 t} |a\rangle \langle c| + H.c. \quad (13)
\end{aligned}
$$

A usual way to account for the effect of the laser phase noise is based on density matrix equations [17, 18], which is convenient if the coefficient matrices commute with each other. Here we follow a somewhat different approach as the usual methods are not easily applied. In particular, we consider the state vector instead of density matrix equations.

The state vector of the three-level atomic system is described by

$$
|\psi\rangle = C_a(t) |a\rangle + C_b(t) e^{i\omega_{ab}t} |b\rangle + C_c(t) e^{i\omega_{ac}t} |c\rangle, \quad (14)
$$

where $C_a(t)$, $C_b(t)$ and $C_c(t)$ are the slowly varying amplitudes. The equations of motion for the amplitudes of states $|a\rangle$ and $|b\rangle$ are

$$
\dot{C}_a(t) = i\Omega_1 e^{-i\Delta_1 t} C_c + i\Omega_p e^{-i\Delta_p t} C_b, \quad (15)
$$

$$
\dot{C}_b(t) = i\Omega_p^* e^{i\Delta_p t} C_a - \gamma C_b, \quad (16)
$$

where $\Delta_1 = \nu_1 - \omega_{ac}$ is the drive field detuning, $\Delta_p = \nu - \omega_{ab}$ is the probe detuning, and γ is the decay rate

from level $|b\rangle$. In order to produce gain for the probe field we set the atoms to be initially in the $|c\rangle$ state. To the lowest order of approximation we can take $C_c \approx 1$ and $C_b \approx 0$. It then follows on integrating Eq. (15), that

$$
C_a(t) = \int_0^t i\Omega_1 e^{-i\Delta_1 t'} C_c(t') dt'. \quad (17)
$$

From Eq. (16) we obtain the formal solution

$$
C_b(t) = \int_0^t i\Omega_p^* e^{i\Delta_p t'} C_a(t') e^{-\gamma(t-t')} dt'. \quad (18)
$$

The off-diagonal density matrix elemment ρ_{ab} is equal to (apart from the phase factor $exp[-i\omega_{ab}t]$)

$$
\begin{aligned}
\langle C_a(t) C_b^*(t) \rangle &= \int_0^t -i\Omega_p e^{-i\Delta_p t'} \langle C_a(t) C_a^*(t') \rangle e^{-\gamma(t-t')} dt' \\
&= \int_0^t -i\Omega_p e^{-i\Delta_p t'} e^{-\gamma(t-t')} dt' \int_0^t iC_c e^{-i\Delta_1 t''} dt'' \times \\
&\times \int_0^{t'} -iC_c^* e^{i\Delta_1 t'''} \langle \Omega_1(t'') \Omega_1^*(t''') \rangle dt'''. \quad (19)
\end{aligned}
$$

In order to consider the effect of phase noise, we can write the drive Rabi frequency as $\Omega_1(t) = \Omega_1 e^{i\phi_1(t)}$. As well known the phase fluctuation of a laser is a Wiener-Levy process, i.e., the random phase with Gaussian statistics performs a Brownian motion.

$$
\begin{aligned}
\langle \phi_1(t) \rangle &= 0, \\
\langle \phi_1(t)\phi_1(t') \rangle &= D_1(t + t' - |t - t'|), \quad (20)
\end{aligned}
$$

where D_1 is the phase induced bandwidth. This gives us the correlation

$$
\langle \Omega_1(t)\Omega_1^*(t') \rangle = |\Omega_1|^2 \left\langle e^{i\phi_1(t) - i\phi_1(t')} \right\rangle = |\Omega_1|^2 e^{-D_1|t-t'|}. \quad (21)
$$

On subsituting from Eq. (21) into Eq.(19) we obtain

$$
\langle C_a(t) C_b^*(t) \rangle \cong \frac{\Omega_p |\Omega_1|^2}{\Delta_0^2} \frac{e^{-i\Delta_p t}}{(\Delta_p - \Delta_1) + i(\gamma + D_1)} \\
+ \text{other frequencies}. \quad (22)
$$

In the last step we used the far detuned condition $\Delta_0 \approx \Delta_1 \approx \Delta_p >> D_1, \gamma$ to ignore the small terms. There are also some terms with other frequencies which do not contribute to the probe susceptibility. We recall that the polarization $P = N\mu_{ab}\rho_{ab} = \chi\epsilon_0 E_p$ where the population matrix element $\rho_{ab} = \langle C_a(t) C_b^*(t) \rangle e^{-i\omega_{ab}t}$. Therefore with both Raman transitions the probe susceptibility under phase noises is

$$
\chi(\Delta_p) = \frac{M_1}{(\Delta_p - \Delta_1) + i(\gamma + D_1)} + \frac{M_2}{(\Delta_p - \Delta_2) + i(\gamma + D_2)}. \quad (23)
$$

This is Eq. (1) if we take $\Gamma_j = \gamma + D_j, (j = 1, 2)$. The inclusion of phase noise effectively increases the width of the gain peaks. From Eqs. (4) we find that larger Γ
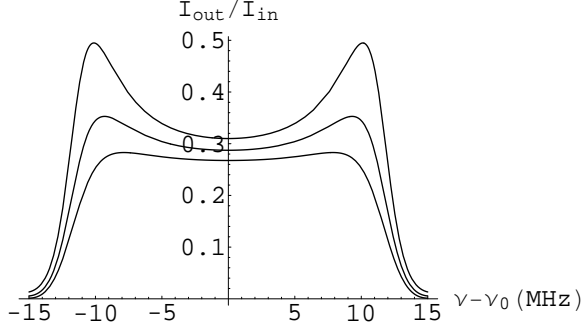
FIG. 3: The transmission of the white light cavity. White light condition is satisfied in all curves. The only difference is that the decay rate increases from lower curve to higher curve.

decreases the magnitude of the dispersion. In order to keep the white light condition we can adjust the param-

eters. For example, we can increase the field intensity to get larger M_j, or alternatively we can use a smaller Δ. Although the dispersion condition can be restored, there is still an impact to the cavity transmission, as shown in Fig. (3). All the three curves are under white light condition with the same parameters except that Γ increases from the lowest to the highest curve. We find that the transmission bandwidth is slightly increased but the curve becomes more uneven, which is not favorable.

Next we consider the effect of amplitude noise, $\Omega_1(t) = \Omega_1 + \delta\Omega_1(t)$. The Gaussian type fluctuation can be described by an Ornstein-Uhlenbeck stochastic process [19]

$$\langle \delta\Omega_1(t) \rangle = 0,$$
$$\langle \delta\Omega_1(t)\delta\Omega_1(t') \rangle = I_{\Omega 1} A_1 e^{-A_1|t-t'|}, \qquad (24)$$

where $I_{\Omega 1}$ is the variance of amplitude fluctuations and A_1 is the amplitude fluctuation induced bandwidth. Again by subsituting from Eq. (24) into Eq.(19) we obtain

$$\langle C_a(t)C_b^*(t) \rangle \cong \frac{\Omega_p|\Omega_1|^2}{\Delta_0^2} \frac{e^{-i\Delta_p t}}{(\Delta_p - \Delta_1) + i\gamma} + \frac{\Omega_p I_{\Omega 1} A_1}{\Delta_0^2} \frac{e^{-i\Delta_p t}}{(\Delta_p - \Delta_1) + i(\gamma + A_1)} + \text{other frequencies}, \qquad (25)$$

$$\chi(\Delta_p) = \frac{M_1}{(\Delta_p - \Delta_1) + i\gamma} + \frac{I_{\Omega 1} A_1}{|\Omega_1|^2} \frac{M_1}{(\Delta_p - \Delta_1) + i(\gamma + A_1)} + \frac{M_2}{(\Delta_p - \Delta_2) + i\gamma} + \frac{I_{\Omega 2} A_2}{|\Omega_2|^2} \frac{M_2}{(\Delta_p - \Delta_2) + i(\gamma + A_2)}. \qquad (26)$$

Similarly we have ignored the small terms in Eq. (25). In Eq. (26) both Raman transitions are included to find the susceptibility under amplitude noise. The net effect are the two additional terms which are similar to the original terms with only different coefficients and γ changed to $\gamma + A_j$. Therefore both the dispersion and gain will increase in magnitudes. Still we can satisfy the white light condition by adjusting the parameters, for example we can decrease the drive field intensity. Similarly we will find the cavity transmission curve becomes uneven since the two additional terms have a larger linewidth $\gamma + A_j$.

IV. CONCLUSION

In this paper we consider the impact of parameter deviations and laser phase and amplitude noises on a bichromatic Raman type white light cavity. We find the dispersion, which needs to satisfy the white light condition,

can be controlled within 10^{-4} under the parameter deviations. Therefore a white light cavity could have 10^4 times broader linewidth compared to an empty cavity at the same finesse.

The phase noise effectively increases the Raman linewidth by the diffusion D, causing a smaller dispersion. The amplitude noise introduces an additional term in the probe susceptibility and makes the dispersion larger. These opposite effects allow us to easily adjust the parameters to satisfy the white light condition. Both noises have the effect of making the transmission curve uneven for the white light cavity.

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